MATH 240: Introduction to Linear Algebra

Summer Session I – 2026

Instructor: Massimo Giulietti, PhD
Credits: 3
Contact Hours: 45

Prerequisites: Calculus I or equivalent
Class Meeting Days & Time: Tuesday and Thursday, 9:15am-12:45pm in the Aula Magna
Office Hours: By appointment after class

Course Type: Standard Course
Course Fee: None

Course Description

This course introduces the fundamental concepts of linear algebra, emphasizing both theoretical foundations and practical applications. Topics include systems of linear equations, matrix algebra, vector spaces, linear transformations, determinants, eigenvalues, eigenvectors, inner products, orthogonality, and least squares solutions. Applications in data science, engineering, and modeling (e.g., Markov chains, data fitting) are integrated to connect theory to real-world problems. The course is designed for students in mathematics, engineering, computer science, and related fields.

Learning Outcomes and Assessment Measures

By the end of this course, students will be able to:

- Solve systems of linear equations using Gaussian elimination and matrix methods (Quizzes, Problem Sets, Midterm, Final).
- Perform matrix operations and compute determinants and inverses (Quizzes, Problem Sets, Midterm, Final).
- Understand and apply concepts of vector spaces, linear transformations, and bases (Problem Sets, Midterm, Final).
- Compute eigenvalues and eigenvectors and apply them to problems like diagonalization (Problem Sets, Midterm, Final).
- Use inner products, orthogonality, and least squares to solve applied problems (Computational Labs, Final).
- Connect linear algebra concepts to applications in data science and engineering (Computational Labs, Problem Sets).

Course Materials

Textbook

Lay, D. C., Lay, S. R., & McDonald, J. J. (2022). *Linear Algebra and Its Applications* (6th ed.). Pearson.

Copies of the required textbook are available for consultation in the library of the Umbra Institute. Students may also choose to purchase the textbook in either print or digital format at their own expense.

A graph notebook will be provided to all students for use throughout the course.

Additional Resources

- A course reader with supplementary notes and practice problems will be provided on Moodle.
- Software: MATLAB, Octave or Python (NumPy) for computational labs, accessible via institutional licenses or free platforms.
- Moodle: Primary location for readings, assignments, and announcements.

Grading

Students are reminded that it is their responsibility to note the dates of exams and other assignments. No alternative exam dates will be offered and professors are not required to give partial credit for any late work (they do so at their discretion: the Institute's default policy is no extensions and a zero for any work turned in late). Students who book travel when they have an exam or other assessment will have to change their plans or accept a zero. Letter grades for student work are based on the following percentage scale:

Letter Grade Range	Numerical Score Equivalent	Student Performance
A A-	93% - 100% 90% - 92%	Exceptional Excellent
B+ B B-	87% -89% 83% - 86% 80% - 82%	Superior
C+ C C-	77% - 79% 73% - 76% 70% - 72%	Satisfactory
D+ D D-	67% - 69% 63% - 66% 60% - 62%	Low Pass
F	59% or less	Fail (no credit)

***<u>Please note</u>**: decimal numerals between 1-4 are rounded down while 5-9 are rounded up: e.g., expect 89.4 to be 89.0 while 89.5 to round up to 90.*

Course Requirements

Attendance (10%): One excused "sick day" allowed. Each unexcused absence reduces the final grade by 4% (up to 10% total). No make-ups for attendance.

Quizzes (10%): Weekly quizzes test foundational skills (e.g., row reduction, matrix inverses). Administered online via Moodle and composed of multiple-choice questions.

Problem Sets (25%): Weekly assignments include theoretical proofs and applied problems (e.g., Markov chains). Completed independently by students outside of class and submitted in hard copy during class.

Computational Labs (15%): Bi-weekly labs involve coding (MATLAB, Octave or Python) for applications such as least squares or eigenvalue computations. Submitted via Moodle.

Midterm Quiz (20%): Covers Weeks 1–2 (systems, matrices, vector spaces). In-class, Week 3.

Final Exam (20%): Comprehensive, covering all topics. Administered in class on the last day of Week 5.

Extension & Submitting Late Work
Umbra Academic Policies apply. Work submitted after deadlines receives a zero, except for one 24-hour extension (email instructor before deadline). No extensions for the final exam.

Attendance Policy
Attendance is expected and mandatory for classroom times and co-curricular activities. <u>The first absence per course due to illness will be considered an excused "sick day" and does not require medical documentation</u>. To receive additional excused absences due to illness, students are required to see a local physician or request a letter from an Institute-approved doctor documenting they should be excused from class for illness.
<u>Unexcused absences will adversely affect a student's academic performance and will result in a reduction of the student's final course grade by 2% per absence up to a maximum of 10%</u>. Excessive unexcused absences may result in a failing grade or disciplinary action. It is the student's responsibility to be aware of the number of absences or late arrivals for each course, and to ask the instructor when in doubt.
If students miss class, they are responsible for obtaining class notes from other students and/or for meeting the professor during office hours. Any work missed in class because of an excused absence may be made up within one week of the return to the class. Any work missed that was a quiz or other test must be made up outside of class time and will, in the interest of intellectual honesty, be a slightly different test than the one given in class.
Legitimate reasons for an excused absence or tardiness includes: death in immediate family, religious observances, illness or injury, local inclement weather, medical appointments that cannot be rescheduled.
Students who request an approved absence to observe a religious holiday must submit a formal request to the Institute's Director within one week after the add/drop period when course schedules, including any field trips, are finalized. No exceptions will be made after this deadline.
Except in the case of medical emergencies, absences are not accepted when tests are scheduled; tests cannot be made up. Furthermore, scheduled times and dates indicated for exams, quizzes, oral presentations, and any other graded assignments cannot be changed for any reason. Even if more sections of the same class are activated, students may only take exams during the scheduled times and dates for the section they are enrolled in.

Tardiness Policy

Students are expected to attend all classes punctually. Any student arriving up to 15 minutes late or leaving up to 15 minutes earlier than the scheduled class end time will be marked as tardy. Each incident of tardiness (late arrivals to or early departures from class) is 0.5% off the final grade. However, should a student arrive more than 15 minutes late or depart more than 15 minutes before the conclusion of the class, it will be recorded as an absence.
Students are also expected to remain in class during the time of instruction except for a reasonable amount of time to use the restroom. Students who leave class and do not return during the class session will receive an unexcused absence or late penalty.

Academic Integrity
All forms of cheating (i.e., copying during exam either from a fellow student or making unauthorized use of notes) and plagiarism (i.e., presenting the ideas or words of another person for academic evaluation without acknowledging the source) will be handled according to the Institute Academic Policy, which can be found in the Umbra Institute Academic Policies and Conduct Guidelines.
Utilizing ChatGPT or other artificial intelligence (AI) tools for the generation of content submitted by a student as their own as part of any assignment for academic credit at the Institute constitutes a form of plagiarism. Should the Institute become aware of a student's use of such platforms and services, the student will be subject to the same consequences and judicial proceedings as are in place for plagiarism (defined above).

Classroom Policy
Umbra Academic Policies apply. Students are expected to follow the policy of the Institute and demonstrate the appropriate respect for the historical premises that the school occupies.

Moodle
Please note that Moodle, not this syllabus, is the ultimate reference for due dates, assignment prompts, and course announcements. It is *the student's responsibility* to check the site regularly to be aware of announcements as well as to see and record all due dates for assignments.

Software Note for Computational Labs: It is not strictly necessary to download and install Octave on your personal computer. To complete and test all the assigned exercises, we recommend using **Octave Online** (https://octave-online.net), a free cloud environment accessible directly from any web browser. The platform allows you to run scripts, view plots, and manage the workspace exactly like the desktop version. For your assignments, you can simply write your code directly online, download the file in `.m` format using the dedicated button, and upload it to Moodle.

Schedule of Topics, Readings, and Assignments

(Each meeting is 3.5 hours with break)

Week 1: Systems of Linear Equations and Matrices

- Meeting 1, part 1: Lecture and examples on introduction to linear systems, Gaussian elimination.
- Meeting 1, part 2:MATLAB/Octave/Python assistance on row operations, reduced row echelon form (assistance: setup, matrix input).
- Meeting 2, part 1: Lecture and examples on matrix operations (addition, scalar multiplication, multiplication).
- Meeting 2, part 2: Lecture, examples, MATLAB/Octave/Python assistance on matrix inverses, solving systems with matrices (assistance: matrix operations).
- Readings: Lay et al., *Linear Algebra and Its Applications* (6th ed.), Sections 1.1–1.5, 2.1–2.2.
- Assignments:

  - Weekly Quiz 1
  - Problem Set 1 (due end of Week 1): Solve linear systems using Gaussian elimination (e.g., 3x3 systems), compute matrix products, find inverses by row reduction.

Week 2: Vector Spaces and Linear Transformations

- Meeting 1, part 1: Lecture and examples on vectors in $\mathbb{R}^n$, linear combinations, span.
- Meeting 1, part 2: Lecture, examples, and MATLAB/Octave/Python on linear independence, bases (assistance: vector operations).
- Meeting 2, part 1: Lecture and examples on vector spaces, subspaces.
- Meeting 2, part 2: Lecture, examples, MATLAB/Octave/Python on linear transformations, matrix representations (assistance: transformation matrices).
- Readings: Lay et al., Sections 4.1–4.4, 5.1–5.2.
- Assignments:
  - Weekly Quiz 2
  - Problem Set 2 (due end of Week 2): Determine linear independence of vectors, find bases for subspaces, compute matrix representations of transformations.
  - Computational Lab 1 (due end of Week 2): Use MATLAB/Octave/Python to check linear independence (e.g., rank with np.linalg.matrix_rank), visualize transformations. Start in assistance sessions (Meetings 2, 4), complete outside. Submit .m or .py file via Moodle.

Week 3: Determinants and Inverses

- Meeting 1, part 1: Lecture and examples on determinants: definition, properties.
- Meeting 1, part 2: Lecture, examples, and MATLAB/Octave/Python on computing determinants, Cramer's rule (assistance: determinant computation).
- Meeting 2, part 1: Lecture and examples on matrix inverses: methods and applications.
- Meeting 2, part 2: Midterm Exam (covers Weeks 1–2), MATLAB/Octave/Python (review determinants), review.
- Readings: Lay et al., Sections 3.1–3.3, 2.3.
- Assignments:
  - Weekly Quiz 3
  - Problem Set 3 (due end of Week 3): Compute determinants (2x2, 3x3), solve systems using Cramer's rule, derive inverse formulas.

Week 4: Eigenvalues and Eigenvectors

- Meeting 1, part 1: Lecture and examples on eigenvalues, eigenvectors: definitions.
- Meeting 1, part 2: Lecture, examples, and MATLAB/Octave/Python on characteristic polynomial, computing eigenvalues (assistance: eigenvalue computation).
- Meeting 2, part 1: Lecture and examples on diagonalization of matrices.
- Meeting 2, part 2: Lecture, examples, MATLAB/Octave/Python on applications (e.g., Markov chains) (assistance: Markov chain simulation).
- Readings: Lay et al., Sections 5.3, 5.5, 7.1.
- Assignments:
  - Weekly Quiz 4
  - Problem Set 4 (due end of Week 4): Find eigenvalues/eigenvectors, diagonalize matrices, analyze Markov chain transitions.

    - Computational Lab 2 (due end of Week 4): Use MATLAB/Octave/Python to compute eigenvalues (e.g., np.linalg.eig), simulate Markov chains. Start in assistance sessions (Meetings 2, 4), complete outside. Submit .m or .py file via Moodle.

Week 5: Inner Products and Orthogonality

- Meeting 1, part 1: Lecture and examples on inner products, norms.
- Meeting 1, part 2: Lecture, examples, and MATLAB/Octave/Python on orthogonality, Gram-Schmidt process (assistance: Gram-Schmidt code).
- Meeting 2, part 1: Lecture, examples, and MATLAB/Octave/Python on least squares, applications (e.g., data fitting) (assistance: least squares problems).
- Meeting 2, part 2: Final Exam (comprehensive, covering all topics), MATLAB/Octave/Python assistance (review least squares code), review (finalize lab submission).
- Readings: Lay et al., Sections 6.1–6.3, 6.7.
- Assignments:
    - Weekly Quiz 5
    - Problem Set 5 (due end of Week 5): Compute inner products, apply Gram-Schmidt process, solve least-squares problems.